1 Problem 12-74

Given: The velocity of a particle is given by $v = (16t^2 i + 4t^3 j + (5t + 2)k)$ m/s where, t is in seconds. If the particle is at the origin when $t = 0$, determine the magnitude of the particle's acceleration when $t = 2$s. Also, what is the x, y, z coordinate position of the particle at this instant.

$$\frac{d\vec{v}}{dt} = \vec{a}$$

$$\vec{a} = (32t\ i + 12t^2\ j + 5\ k)$$

$$\frac{d\vec{s}}{dt} = \vec{v}$$

$$d\vec{s} = \vec{v}\ dt$$

$$\int d\vec{s} = \int (16t^2 i + 4t^3 j + (5t + 2)k)dt$$

$$\vec{s} = \frac{16}{3}t^3 i + t^4 j + (\frac{5}{2}t^2 + 2t)k + C$$

$$\vec{a}(2) = (64i + 48j + 5k)$$

$$\|\vec{a}(2)\| = 5\sqrt{257}$$

$$\|\vec{a}(2)\| = 80.2[m/s^2]$$

$$\vec{s}(2) = (128/3i + 16j + 14k)$$

Note: Defaulting to 3 sig figs.

2 Problem 12-78

Given: Pegs A and B are restricted to move in the elliptical slots due to the motion of the slotted link. If the link moves with a constant speed of 10m/s, determine the magnitude of the velocity and acceleration of peg A when x = 1m.

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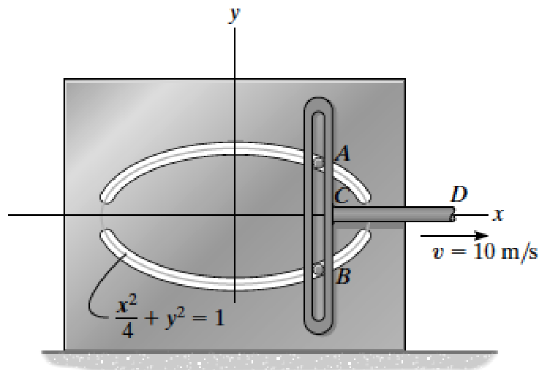


$$\frac{dx}{dt} = 10[m/s]$$

$$\frac{d}{dt}\left(\frac{x^2}{4} + y^2\right) = \frac{d}{dt}(1)$$

$$\frac{1}{2}x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

solving for $\frac{dy}{dt}$

$$\frac{dy}{dt} = \frac{x}{4y}\frac{dx}{dt}$$

$$\frac{dy}{dt} = -\frac{10x}{4y}$$

$$\vec{v} = \left\{\frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}\right\}$$

$$\vec{v} = \left\{10\hat{i} - \frac{10x}{4y}\hat{j}\right\}$$

Solving for $\vec{a}$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d(10)}{dt}\hat{i} - \frac{d}{dt}\left(\frac{10x}{4y}\right)\hat{j}$$

applying the chain rule and substituting previous solutions back in yields:

$$\vec{a} = \frac{d\vec{v}}{dt} = 0\hat{i} - \left(\frac{25}{y} - \frac{25x^2}{4y^3}\right)\hat{j}$$

when x equals 1:

$$y = \sqrt{.75} = \sqrt{1 - \frac{1^2}{4}}$$

$$\frac{dy}{dt} = \frac{10}{4\sqrt{.75}}$$

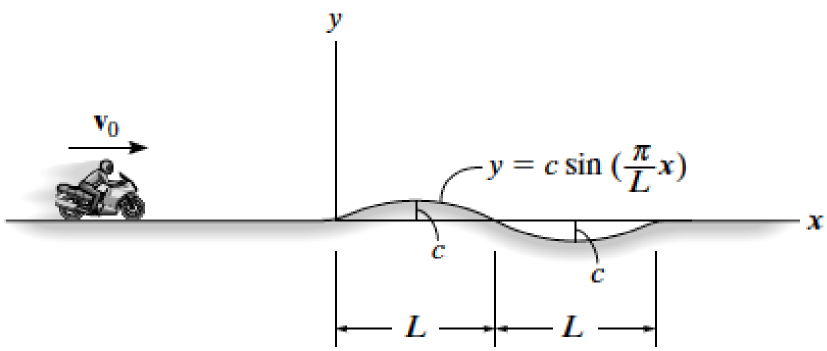
$$\|\vec{v}(1,4)\| = \|10i - \frac{10}{4\sqrt{.75}}j\| = \boxed{10.4 \text{ [m/s]}}$$

$$\|\frac{d\vec{v}}{dt}\| = \|0\hat{i} - \left(\frac{25}{\sqrt{.75}} - \frac{25(1)^2}{4\sqrt{(}.75)^3}\right)\hat{j}\| = \boxed{38.5 \text{ [m/(s*s)]}}$$

Note: Defaulting to 3 sig figs.

3 Problem 12-86

Given: The motorcycle travels with a constant speed v_0 along the path that, for a short distance, takes the form of a sine curve. Determine the x and y components of its velocity at any instant on the curve.



$$v_0 = \sqrt{\frac{dx}{dt}^2 + \frac{dx}{dt}^2} \tag{1}$$

$$\frac{dy}{dt} = v_y = \frac{d}{dt}\left(c \sin\left(\frac{\pi}{L} x\right)\right) = c\left(\cos\left(\frac{\pi}{L} x\right) \frac{\pi}{L} \frac{dx}{dt}\right) \tag{2}$$

substituting this into the origional equation 1 yields:

$$v_0 = \sqrt{\frac{dx}{dt}^2 \left(1 + \left(\cos\left(\frac{\pi}{L} x\right) \frac{\pi c}{L}\right)^2\right)} \tag{3}$$

solving this for $\frac{dx}{dt}$ yields:

$$\frac{dx}{dt} = v_0 \sqrt{1 + \left(\cos\left(\frac{\pi}{L} x\right) \frac{\pi c}{L}\right)^2} \tag{4}$$

substituting equation 4 into equation 2 yields:

$$\frac{dy}{dt} = c\left(\cos\left(\frac{\pi}{L} x\right) \frac{\pi}{L}\right) v_0 \sqrt{1 + \left(\cos\left(\frac{\pi}{L} x\right) \frac{\pi c}{L}\right)^2} \tag{5}$$